

# Conformal Sigma Models for a Class of $T^{p,q}$ Spaces

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## Abstract

We consider a 2-d conformal theory based on  $\frac{G \times G'}{H}$  coset sigma model introduced by Guadagnini, Martellini and Mintchev. It is shown that in the case of  $\frac{SU(2) \times SU(2)}{U(1)}$  the metric of the corresponding background is of  $T^{p,q}$  coset space form (but is not an Einstein one). Similar interpretation is possible for the Lorentzian coset space  $W_{4,2} = \frac{SL(2,R) \times SL(2,R)}{U(1)}$ . The resulting 10-d homogeneous space metric on  $W_{4,2} \times T^{p,q}$  supplemented with 2-form field gives a critical NS-NS superstring background with conformal sigma model interpretation.

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# 1 Introduction

Metrics of physically interesting backgrounds usually have large amount of global symmetry. One set of examples are black hole metrics with rotational symmetry, and another are symmetric spaces  $AdS_n \times S^n$  supported by R-R antisymmetric tensor backgrounds. At the same time, string solutions which have known 2-d CFT interpretation, like gauged WZW models, have associated space-time metrics with very few or no global symmetries. It is of interest to look for new examples of conformal sigma models related to metrics on symmetric spaces.

Special symmetric spaces that were recently discussed in connection with AdS/CFT correspondence are  $T^{p,q}$  spaces. These are cosets of the form  $T^{p,q} = [SU(2) \times SU(2)]/U(1)$  with the integers  $p$  and  $q$  determining the embedding of the  $U(1)$  subgroup. Their metric is [1, 2, 3]

$$ds^2 = \lambda_1^2(d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2) + \lambda_2^2(d\theta_2^2 + \sin^2 \theta_2 d\phi_2^2) + \lambda^2(d\psi + p \cos \theta_1 d\phi_1 + q \cos \theta_2 d\phi_2)^2. \quad (1.1)$$

A particular case of  $p = q = 1$  relevant for discussions of AdS supergravity solutions preserves part of supersymmetry, and with  $\lambda_1^2 = \lambda_2^2 = 1/6$  and  $\lambda^2 = 1/9$  its metric is an Einstein space one.

Below we shall show that certain symmetric metrics on  $T^{p,q}$  spaces can be interpreted as parts of NS-NS string backgrounds associated with a class of conformal coset sigma models proposed by Guadagnini, Martellini and Mintchev (GMM) [4, 5]. Though these metrics supported by NS-NS 2-form field are not Einstein ones (in contrast to the  $T^{1,1}$  example studied [6] in connection with AdS/CFT correspondence), they may still turn out to be of some interest.

The conformal sigma model with  $T^{p,q}$  metric should be supplemented by another one to balance the central charge. One example of a critical  $D = 10$  string model has the metric of the form  $W_{4,2} \times T^{1,q}$ , where  $W_{4,2} = SO(2, 2)/SO(2) = [SL(2, R) \times SL(2, R)]/U(1)$ . An Einstein representative in the class of metrics on  $W_{4,2}$  was discussed in [7] as a generalization of  $AdS_5$ .

In section 2 we shall review the GMM construction [4] and its interpretation as a coset CFT [5]. We shall also comment on its relation to a class of gauged WZW models as discussed in [8]. In section 3 we shall consider a GMM model that leads to a  $T^{p,q}$  metric. Section 4 is devoted to explicit check of conformal invariance of the corresponding sigma model at the one- and two-loop levels.

## 2 The Guadagnini-Martellini-Mintchev Model

The starting point is the WZW action [9]

$$I(U; n) = \frac{n}{8\pi} \int_{\partial M} d^2x \, Tr(\partial_\mu U \partial^\mu U^{-1}) + \frac{n}{12\pi} \int_M d^3y \, \epsilon^{ijk} \, Tr(U^{-1} \partial_i U U^{-1} \partial_j U U^{-1} \partial_k U), \quad (2.1)$$

where  $U$  is an element of the group  $G$  and  $n$  is the level of the associated affine Kac-Moody algebra. The property of the WZW model that is used in the GMM construction is that under an arbitrary variation of the group element  $\delta U$  the WZW action changes by

$$\delta I(U; n) = \frac{n}{4\pi} \int d^2x \, Tr[U^{-1} \delta U (\eta_{\mu\nu} - \epsilon_{\mu\nu}) \partial_\mu (U^{-1} \partial_\nu U)]. \quad (2.2)$$

This variation can be written also as

$$\delta I(U; n) = \frac{n}{4\pi} \int d^2x \operatorname{Tr}[\delta U U^{-1}(\eta_{\mu\nu} + \epsilon_{\mu\nu})\partial_\mu(\partial_\nu U U^{-1})], \quad (2.3)$$

or as  $\int d^2z \operatorname{Tr}[U^{-1}\delta U \partial_z(U^{-1}\partial_{\bar{z}}U)] = \int d^2z \operatorname{Tr}[\delta U U^{-1}\partial_{\bar{z}}(\partial_z U U^{-1})]$ . From these variations one can read off the currents associated with the symmetry  $U \rightarrow \Omega(z)U\bar{\Omega}^{-1}(\bar{z})$  [9].

Consider the variation of the WZW model under the following gauge transformation

$$U \rightarrow UR(\Omega^{-1}), \quad (2.4)$$

where  $R$  is a representation of a subgroup  $H \subset G$  and  $\Omega \in H$ . Under infinitesimal transformations  $\Omega(x) = 1 + \omega(x)$  the WZW action transforms as (2.2)

$$\delta I(U; n) = -\frac{n}{4\pi} \int d^2 \operatorname{Tr}[R(\omega)\partial^\mu(U^{-1}\partial_\mu U - \epsilon^{\mu\nu}U^{-1}\partial_\nu U)], \quad (2.5)$$

where we set  $R(\Omega^{-1}) = 1 - R(\omega) + \dots$ . In order to cancel this “classical anomaly” GMM suggested to introduce another field  $V$  belonging to a group  $G'$  whose action has similar anomalous transformation property under  $H$ . It is assumed that the same  $H$  is a subgroup of both  $G$  and  $G'$  so that the class of resulting coset models is rather special. Let  $V \in G'$  and  $R'$  be a representation of  $H \subset G'$  acting on  $V$  according to

$$V \rightarrow R'(\Omega)V. \quad (2.6)$$

Using Eq. (2.3) we get for the variation of the WZW action  $I(V; m)$  similar to (2.1)

$$\delta I(V; m) = \frac{m}{4\pi} \int d^2 \operatorname{Tr}[R'(\omega)\partial^\mu(\partial_\mu V V^{-1} + \epsilon_{\mu\nu}\partial^\nu V V^{-1})]. \quad (2.7)$$

One can then check that the model

$$\begin{aligned} I_{GMM} &= I(U; n) + I(V; m) + I_{int}(U, V; k), \\ I_{int}(U, V; k) &= -\frac{k}{2\pi} \int d^2x \left[ \operatorname{Tr}(R_\alpha U^{-1}\partial_\mu U) \operatorname{Tr}(R'_\alpha \partial^\mu V V^{-1}) \right. \\ &\quad \left. + \epsilon^{\mu\nu} \operatorname{Tr}(R_\alpha U^{-1}\partial_\mu U) \operatorname{Tr}(R'_\alpha \partial_\nu V V^{-1}) \right] \end{aligned} \quad (2.8)$$

is gauge invariant for

$$\begin{aligned} n &= kr', & m &= kr, \\ \operatorname{Tr} R_\alpha R_\beta &= r\delta_{\alpha\beta}, & \operatorname{Tr} R'_\alpha R'_\beta &= r'\delta_{\alpha\beta}, \end{aligned} \quad (2.9)$$

where, as in [4], the generators of the Lie algebras of  $G$  and  $G'$  are  $\{R_i\} = \{R_I, R_\alpha\}$  and  $\{R'_a\} = \{R'_A, R'_\alpha\}$ , where  $R_\alpha$  and  $R'_\alpha$  correspond to the Lie algebra of subgroup  $H$ . The one-loop finiteness of this model was checked in [4] and finiteness at the two-loop level was checked in [10]. The conformal field theory defined by this sigma model was discussed in ref. [5], which found the current algebra and the Virasoro algebra with a central charge value coinciding with that of the GKO construction [11, 12] for the coset  $(G \times G')/H$ .

Let us briefly review the conformal structure of the GMM model. The variation of the action (2.8) with respect to  $U$  and  $V$  yields the following equations of motion

$$\partial_{\bar{z}} J_z^i = 0, \quad \partial_z J_{\bar{z}}^a = 0, \quad (2.10)$$

where

$$\begin{aligned} J_z^i &= (\partial_z U U^{-1})^i + \frac{1}{kr'} (U R_\alpha U^{-1})^i \text{Tr}(R'_\alpha \partial_z V V^{-1}), \\ J_{\bar{z}}^a &= -(V^{-1} \partial_{\bar{z}} V)^a + \frac{1}{kr} (V^{-1} R'_\alpha V)^a \text{Tr}(R_\alpha U^{-1} \partial_{\bar{z}} U). \end{aligned} \quad (2.11)$$

The form of the equations of motion and currents suggests, by analogy with the WZW model, the existence of two copies of affine algebras [5]. Introducing

$$K_z^a = (\partial_z V V^{-1})^a, \quad K_{\bar{z}}^i = -(U^{-1} \partial_{\bar{z}} U)^i, \quad (2.12)$$

one can write the components of the classical energy-momentum tensor as

$$\begin{aligned} T_z &= \frac{1}{kr'} J_z^i J_z^i + \frac{1}{kr} K_z^A K_z^A, \\ T_{\bar{z}} &= \frac{1}{kr} J_{\bar{z}}^a J_{\bar{z}}^a + \frac{1}{kr'} K_{\bar{z}}^I K_{\bar{z}}^I. \end{aligned} \quad (2.13)$$

The analysis of this bosonic model at the quantum level reveals that the central charge is [5]

$$c_{GMM} = c(G, kr') + c(G', kr) - c(H, 2kr r'), \quad (2.14)$$

with  $c(G, n) = n \dim G / [n + 2c_V(G)]$ , where  $c_V(G) \delta_{ab} = f_{acd} f_{bcd}$ . The quantum energy-momentum tensor is of the same form as the classical one but with rescaled coefficients [5, 13]

$$\begin{aligned} T_z &= \frac{1}{kr' + 2c_V(G)} : J_z^i J_z^i : + \frac{1}{kr + 2c_V(G')} : K_z^A K_z^A :, \\ T_{\bar{z}} &= \frac{1}{kr + 2c_V(G')} : J_{\bar{z}}^a J_{\bar{z}}^a : + \frac{1}{kr' + 2c_V(G)} : K_{\bar{z}}^I K_{\bar{z}}^I :. \end{aligned} \quad (2.15)$$

The expressions in the supersymmetric case are similar, with levels shifted ( $kr' + 2c_V(G) \rightarrow kr'$ , etc) as in the (gauged) WZW model case (see, e.g., [14] and refs. there).

Let us note also that the GMM model can be represented as a kind of generalized gauged WZW model which is free of anomalies and upon elimination of the 2-d gauge fields reduces to the GMM action. Introducing non-dynamical 2-d gauge fields  $A$  and  $B$  one may consider the action

$$\begin{aligned} \hat{I}_{GMM} &= I(U; n) + I(V; m) + I_{int}(U, V, A, B; k), \\ I_{int}(U, V; k) &= -\frac{k}{4\pi} \int d^2 z \left[ \text{Tr}(R_\alpha A_{\bar{z}}) \text{Tr}(R'_\alpha \partial_z V V^{-1}) - \text{Tr}(R'_\alpha B_z) \text{Tr}(R_\alpha U^{-1} \partial_{\bar{z}} U) \right. \\ &\quad \left. + \text{Tr}(R_\alpha A_{\bar{z}}) \text{Tr}(R'_\alpha B_z) \right], \end{aligned} \quad (2.16)$$

which is invariant under the following gauge transformations:

$$\begin{aligned} \delta U &= -U \omega, & \delta V &= \omega V, \\ \delta B_i &= -\partial_i \omega - [B_i, \omega], & \delta A_i &= -\partial_i \omega - [A_i, \omega]. \end{aligned} \quad (2.17)$$

Integrating out the gauge fields gives back the GMM action Eq.(2.8).<sup>2</sup> In the standard diagonal vector gauged WZW model the gauge action is  $g \rightarrow hgh^{-1}$ . The GMM model may be interpreted as a gauged WZW model defined on the product group  $G' \times G$ , with the gauged subgroup acting as  $(V, U) \rightarrow (hV, Uh^{-1})$ , i.e. it may be viewed as a non-anomalous “sum” of right and left gauged [8] WZW models.

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<sup>2</sup>Note that, in contrast to what happens in the usual gauged WZW models [15, 16], integrating out the gauge fields gives trivial determinant, i.e. does not produce a non-constant dilaton coupling.

### 3 $T^{p,q}$ and $W_{4,2}$ metrics from GMM model

Let us consider the GMM model for  $G = SU(2)$ ,  $G' = SU(2)$ , and  $H = U(1)$ . The  $SU(2)$  group elements are parametrized according to

$$\begin{aligned} U &= \exp(i\phi_1\sigma_3) \exp(i\theta_1\sigma_2) \exp(i\psi_1\sigma_3), \\ V &= \exp(i\phi_2\sigma_3) \exp(i\theta_2\sigma_2) \exp(i\psi_2\sigma_3). \end{aligned} \quad (3.1)$$

The gauge action of the  $U(1)$  subgroup is defined by

$$\psi_1 \rightarrow \psi_1 - p\epsilon(z, \bar{z}), \quad \phi_2 \rightarrow \phi_2 + q\epsilon(z, \bar{z}). \quad (3.2)$$

This corresponds to gauging the subgroup generated by  $i(q\sigma_3^L - p\sigma_3^R)$ . Consider the sum of the two WZW models on  $SU(2)$  with levels  $k_1$  and  $k_2$  and the GMM interaction term (2.8) with coefficient  $k_3$

$$\begin{aligned} I &= \frac{1}{4\pi} \int d^2x \left[ k_1 \left( \partial_\mu \theta_1 \partial^\mu \theta_1 + \partial_\mu \phi_1 \partial^\mu \phi_1 + \partial_\mu \psi_1 \partial^\mu \psi_1 + \cos(2\theta_1) \partial_\mu \phi_1 \partial_\nu \psi_1 (\eta^{\mu\nu} + \epsilon^{\mu\nu}) \right) \right. \\ &+ k_2 \left( \partial_\mu \theta_2 \partial^\mu \theta_2 + \partial_\mu \phi_2 \partial^\mu \phi_2 + \partial_\mu \psi_2 \partial^\mu \psi_2 + \cos(2\theta_2) \partial_\mu \phi_2 \partial_\nu \psi_2 (\eta^{\mu\nu} + \epsilon^{\mu\nu}) \right) \\ &+ k_3 \left( \cos(2\theta_1) \partial_\mu \phi_1 + \partial_\mu \psi_1 \right) \left( \cos(2\theta_2) \partial_\nu \psi_2 + \partial_\nu \phi_2 \right) (\eta^{\mu\nu} + \epsilon^{\mu\nu}) \Big]. \end{aligned} \quad (3.3)$$

For the action to be invariant under (3.2) one needs to impose the following algebraic constraints:

$$k_1 p = k_3 q, \quad k_2 q = k_3 p. \quad (3.4)$$

Multiplying these equations we get that

$$k_3 = \sqrt{k_1 k_2}, \quad p/q = \sqrt{k_2/k_1}. \quad (3.5)$$

Fixing the gauge as  $\phi_2 = 0$  one gets a background whose metric is of the (non-Einstein)  $T^{1,Q}$  type

$$ds^2 = k[d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2 + Q^2(d\theta_2^2 + \sin^2 \theta_2 d\phi_2^2) + (d\psi + \cos \theta_1 d\phi_1 + Q \cos \theta_2 d\phi_2)^2], \quad (3.6)$$

where we have rescaled all variables by  $1/2$ , renamed  $\psi_2 \rightarrow \phi_2$ ,  $\psi_1 \rightarrow \psi$  and introduced

$$Q = p/q = \sqrt{k_2/k_1}, \quad k = k_1. \quad (3.7)$$

The background also includes the antisymmetric field

$$B_{\phi_1 \psi} = k \cos \theta_1, \quad B_{\phi_1 \phi_2} = kQ \cos \theta_1 \cos \theta_2, \quad B_{\phi_2 \psi} = -kQ \cos \theta_2. \quad (3.8)$$

The coefficients in front of the different terms in the action are dictated by gauge invariance of the total action and can not be re-adjusted.

Fixing the gauge in the original variables as  $\psi_1 = 0$  one ends up with a metric of the type  $T^{Q^{-1},1}$ . More generally, imposing  $\psi_1 = \Lambda \phi_2$  as gauge fixing condition is equivalent, at the level of the metric, to the rescaling  $\psi \rightarrow (Q + \Lambda)\psi$ . Undoing this rescaling takes the resulting background into that of  $T^{1,Q}$  presented above.

The central charge of this model is (see Eq. (2.14))

$$c = \frac{3k_1}{k_1 + 2} + \frac{3k_2}{k_2 + 2} - 1 = \frac{3k}{k + 2} + \frac{3kQ^2}{kQ^2 + 2} - 1, \quad (3.9)$$

and reduces to 5 in the semiclassical limit ( $k \rightarrow \infty$ ). In order to get a critical string background we need to add another model to compensate for the central charge deficit. One natural possibility is to consider a Lorentzian version of  $T^{p,q}$ . Namely, consider the GMM model for  $G = SL(2, R)$ ,  $G' = SL(2, R)$  and  $H = U(1)$ . The group elements are parametrized as

$$\begin{aligned} U &= \exp(i\phi_1\sigma_3) \exp(r_1\sigma_2) \exp(i\psi_1\sigma_3), \\ V &= \exp(i\phi_2\sigma_3) \exp(r_2\sigma_2) \exp(i\psi_2\sigma_3) \end{aligned} \quad (3.10)$$

and by analogy with the  $SU(2) \times SU(2)$  case we define the following action of the subgroup

$$\psi_1 \rightarrow \psi_1 - p\epsilon(z, \bar{z}), \quad \phi_2 \rightarrow \phi_2 + q\epsilon(z, \bar{z}). \quad (3.11)$$

Following the same steps as above we end up with the following background

$$\begin{aligned} ds^2 &= k \left[ dr_1^2 + \sinh^2 r_1 d\phi_1^2 + Q^2 (dr_2^2 + \sinh^2 r_2 d\phi_2^2) \right. \\ &\quad \left. - (dt + \cosh r_1 d\phi_1 + Q \cosh r_2 d\phi_2)^2 \right], \end{aligned} \quad (3.12)$$

$$B_{\phi_1 t} = k \cosh r_1, \quad B_{\phi_1 \phi_2} = kQ \cosh r_1 \cosh r_2, \quad B_{\phi_2 t} = -kQ \cosh r_2. \quad (3.13)$$

This metric (which may be viewed as a formal analytic continuation of the above  $T^{1,Q}$  metric (3.5)) belongs to a class of noncompact versions of Stiefel manifold, and corresponds to  $W_{4,2} = SO(2, 2)/SO(2)$ . The parameters  $k, Q$  here are the same as above, so that the deficit of the central charge cancels just as in the  $SL(2, R) \times SU(2)$  WZW model (to the leading approximation in the bosonic case, and exactly in the supersymmetric case).

One can check that the total  $d = 10$  background we constructed is not supersymmetric. This is in contrast to what happens in the case of  $W_{4,2} \times T^{1,1}$  Freund-Rubin type solution of IIB supergravity supported by 5-form field [7], where the metrics on the cosets are chosen to be the Einstein ones, and 1/4 of supersymmetry is preserved.

## 4 Check of conformal invariance

It is easy to check that the one-loop conformal invariance equations  $R_{\mu\nu} - \frac{1}{4}H_{\mu\lambda\rho}H_{\nu}{}^{\lambda\rho} = 0$  [17, 18] are satisfied. The components of the Ricci tensor and the scalar curvature of the  $T^{1,Q}$  sector are

$$\begin{aligned} R_{\theta_1\theta_1} &= \frac{1}{2}, \quad R_{\phi_1\phi_1} = \frac{1}{2Q^2}(Q^2 + \cos^2 \theta_1), \quad R_{\phi_1\psi} = \frac{Q^2 + 1}{2Q^2} \cos \theta_1, \\ R_{\theta_2\theta_2} &= \frac{1}{2}, \quad R_{\phi_2\phi_2} = \frac{1}{2}(1 + Q^2 \cos^2 \theta_2), \quad R_{\phi_2\psi} = \frac{Q^2 + 1}{2Q} \cos \theta_2, \\ R_{\psi\psi} &= \frac{Q^2 + 1}{2Q^2}, \quad R_{\phi_1\phi_2} = \frac{Q^2 + 1}{2Q} \cos \theta_1 \cos \theta_2, \quad R = \frac{3}{2} \frac{Q^2 + 1}{kQ^2}, \end{aligned} \quad (4.1)$$

and similar expressions are found for  $W_{4,2}$ . The total scalar curvature of is *zero* since  $R(W_{4,2}) = -R(T^{1,Q}) = -3(Q^2 + 1)/(2kQ^2)$ .

The two-loop  $\beta$ -function for the  $G_{\mu\nu} + B_{\mu\nu}$  coupling of the bosonic sigma model is (assuming a specific scheme, see [19, 20]):

$$\beta_{\mu\nu} = \alpha' \hat{R}_{\mu\nu} + \frac{\alpha'^2}{2} \left[ \hat{R}^{\alpha\beta\gamma}{}_{\nu} \hat{R}_{\mu\alpha\beta\gamma} - \frac{1}{2} \hat{R}^{\beta\gamma\alpha}{}_{\nu} \hat{R}_{\mu\alpha\beta\gamma} + \frac{1}{2} \hat{R}_{\alpha\mu\nu\beta} (H^2)^{\alpha\beta} \right] + O(\alpha'^3), \quad (4.2)$$

where  $\hat{R}_{\alpha\beta\gamma\delta}$  is the Riemann tensor for the generalized connection  $\hat{\Gamma}^\lambda{}_{\mu\nu} = \Gamma^\lambda{}_{\mu\nu} - \frac{1}{2} H^\lambda{}_{\mu\nu}$ . In this scheme a parallelizable manifold having  $\hat{R}_{\alpha\beta\gamma\delta} = 0$  (e.g. a group space) automatically satisfies the two-loop conformal invariance condition. For the background we are discussing the tensor  $\hat{R}_{\alpha\beta\gamma\delta}$  is non-vanishing; e.g., the generalized curvature of the  $T^{1,Q}$  metric is

$$\hat{R}_{\theta_2\phi_2\theta_1\phi_1} = -kQ \sin \theta_1 \sin \theta_2. \quad (4.3)$$

One can check, however, that the beta-function (4.2) still vanishes.<sup>3</sup> Like target space backgrounds appearing in the case of gauged WZW models [15, 21], these backgrounds, though not parallelizable, define conformal sigma models.

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<sup>3</sup>Note again that the one-loop beta function equal to the generalized Ricci tensor  $\hat{R}_{\mu\nu}$  vanishes since  $g^{\theta_1\theta_2} = g^{\phi_2\phi_1} = g^{\theta_2\phi_1} = g^{\theta_1\phi_2} = 0$ .

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